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I. Sc. class - XI
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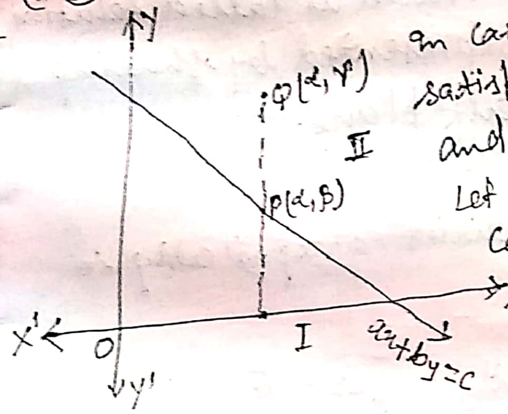
Topic: Linear inequalities.

Graphical Solution of Linear Inequalities in Two variables

Let us consider the line

$$ax+by=c, \; a\neq 0, \; b\neq 0 \quad \text{—— I}$$

Here, there are three cases arise
(i) $ax+by=c$ (ii) $ax+by>c$ (iii) $ax+by<c$



In case I, it is clear that all points (x,y) satisfying (i) lie on the line it represents and conversely. Let us consider case (ii), Let us first assume that $b>0$. Let us consider a point $P(\alpha, \beta)$ on the line $ax+by=c$, $b>0$, so that $a\alpha+b\beta=c$. Take a arbitrary point (α, γ) in the half plane II.

Now, from figure,

$$\gamma > \beta$$
$$\Rightarrow b\gamma > b\beta \Rightarrow a\alpha+b\gamma > a\alpha+b\beta$$
$$\Rightarrow a\alpha+b\gamma > c$$

i.e. $Q(\alpha,\gamma)$ satisfies the inequality $ax+by>c$

Thus, all the points lying in the half plane II above the line $ax+by=c$ satisfies the inequality $ax+by>c$.

Conversely, let (α,β) be a point on line $ax+by=c$ and an arbitrary point $Q(\alpha,\gamma)$ satisfying

$$ax+by>c$$

so that $a\alpha+b\gamma>c$

$$\Rightarrow a\alpha+b\gamma > a\alpha+b\beta$$

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$\Rightarrow \gamma > \beta$ (as $b>0$)

This means that the point (α, γ) lies in the half plane II.

Thus, any point in the half plane II satisfies $ax+by>c$, and conversely any point satisfying the inequality $ax+by>c$ lies in half plane II.

In case $b<0$, we can similarly prove that any point satisfying $ax+by>c$ lies in the half plane I, and conversely.

Hence, we deduce that all points satisfying $ax+by>c$ lies in one of the half planes II or I according as $b>0$ or $b<0$, and conversely.

Thus, graph of the inequality $ax+by>c$ will be one of the half plane which is called solution region and ~~respectively~~ represented by shading in the corresponding half plane.

Exercise 6.2

Solve the following inequalities graphically in two-dimensional plane.

1. $x+y<5$

Soln: We have the given inequality $x+y<5$ ——— (i)

Here, there are four steps arise.

Step I let us consider the inequation as a strict eqn

i.e. $x+y=5$ when $x=5$, then $y=0$

when $x=0$, then $y=5$

Step II

x	5	0	
y	0	5	

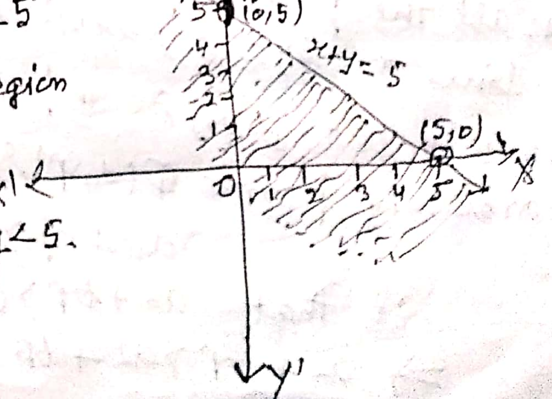
Step III Plot the graph using the above table.

Step IV putting $x=0, y=0$ then by (i), we get

$0+0<5, \quad 0<5$

Which is true, so shaded region will be towards the origin.

Thus, shaded region shows the inequality $x+y<5$.



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Q.11 To find the condition that the line $lx+my+n=0$ may touch the conic $S\equiv ax^2+by^2+2hxy+2gx+2fy+c=0$

Soln The line $lx+my+n=0$ —— I
will touch the conic
$$S\equiv ax^2+by^2+2hxy+2gx+2fy+c=0 \quad \text{—— II}$$
if (I) intersects II at coincident points.

Then by I, $lx+my+n=0 \Rightarrow my=-(lx+n) \Rightarrow y=-\frac{lx+n}{m}$

Putting it in II

$$\cancel{ax^2+by^2+2hx\left(-\frac{lx+n}{m}\right)+b\left(-\frac{lx+n}{m}\right)}$$

$$ax^2+b\left(-\frac{lx+n}{m}\right)^2+2hx\left(-\frac{lx+n}{m}\right)+2gx+2f\left(-\frac{lx+n}{m}\right)+c=0$$

$$\Rightarrow x^2(am^2-2hlm+bl^2)+2x(bln-hnm-flm+gm^2)$$
$$+(bn^2-2fmn+cm^2)=0 \quad \text{—— III}$$

I will touch II if III has equal roots.

∴ The condition for tangency is discriminant $=0$

$$\Rightarrow 4(bln-hnm-flm+gm^2)^2-4(am^2-2hlm+bl^2)$$
$$\times(bn^2-2fmn+cm^2)=0$$

$$\Rightarrow (bln-hnm-flm+gm^2)^2=(am^2-2hlm+bl^2)\times(bn^2-2fmn+cm^2)$$

which is the required condition.

Q.12. What conic section is represented by
$$2x^2+3y^2-4x-12y+13=0\ ?$$
Find the centre, axes and eccentricity.

Soln Let us compare the given conic with the general equation of a conic given by
$$ax^2+by^2+2hxy+2gx+2fy+c=0$$
Let the given conic be written as
$$F\equiv 2x^2+3y^2-4x-12y+13=0$$
In this case, $a=2,\ h=0,\ b=3$

p.12 ans proof [illegible] (rest part of question 12)

$\therefore h^2 - ab = -2\times 3 = -6 < 0$. $\therefore$ The conic is an ellipse.

Again, $\frac{\partial F}{\partial x} = 4x - 4$ and $\frac{\partial F}{\partial y} = 6y - 12$

The centre is the point given by solving the equations

$$\frac{\partial F}{\partial x} = 0 \text{ and } \frac{\partial F}{\partial y} = 0 \text{ simultaneously.}$$

~~Simultaneously.~~

We thus get, $4x - 4 = 0$ and $6y - 12 = 0$

$$\Rightarrow 4x = 4 \quad \text{and } 6y = 12$$

$$\Rightarrow x = \frac{4}{4} = 1 \text{ and } y = \frac{12}{6} = 2$$

$$\Rightarrow x = 1 \quad \text{and } y = 2.$$

$\therefore$ The centre is the point $(1, 2)$

Now, we change the origin to the centre $(1, 2)$.

The new equation of the conic becomes

$$2(x+1)^2 + 3(y+2)^2 - 4(x+1) - 12(y+2) + 13 = 0$$

$$\Rightarrow 2(x^2 + 2x + 1) + 3(y^2 + 4y + 4) - 4(x+1) - 12(y+2) + 13 = 0$$

$$\Rightarrow 2x^2 + 3y^2 = 1 \Rightarrow \frac{x^2}{\frac{1}{2}} + \frac{y^2}{\frac{1}{3}} = 1 \Rightarrow \frac{x^2}{\alpha^2} + \frac{y^2}{\beta^2} = 1.$$

The semimajor axis $\alpha = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$ } ——— I

and the semiminor axis $\beta = \sqrt{\frac{1}{3}} = \frac{1}{\sqrt{3}}$.

The eccentricity is given by $\beta^2 = \alpha^2(1 - e^2)$

$$\Rightarrow \beta^2 = \alpha^2 - \alpha^2 e^2$$

$$\Rightarrow \alpha^2 e^2 = \alpha^2 - \beta^2$$

$$\Rightarrow e^2 = \frac{\alpha^2 - \beta^2}{a^2}$$

$$\Rightarrow e^2 = \frac{\frac{1}{2} - \frac{1}{3}}{\frac{1}{2}} \quad \{\text{From I}$$

$$\Rightarrow e^2 = \frac{\frac{3-2}{6}}{\frac{1}{2}} = \frac{\frac{1}{6}}{\frac{1}{2}}$$

$$\Rightarrow e^2 = \frac{1}{\cancel{6}_3} \times \frac{\cancel{2}}{1}$$

$$\Rightarrow e^2 = \frac{1}{3}$$

$$\therefore e = \frac{1}{\sqrt{3}}$$

Ans